1. INTRODUCTION

Stimulated Brillouin scattering (SBS) has been shown to be an effective method of compressing laser pulses.\(^1\)\(^-\)\(^7\) Previously, the equations of SBS by backward-wave amplification has been solved in two regimes:

- the transient regime,\(^2\)\(^-\)\(^4\) defined as the regime where the partial time derivatives of first order are considered and the pump pulse is short compared with the phonon lifetime;
- the steady-state regime,\(^1\) where all time derivatives are set equal to zero and the pump pulse is long compared with the phonon lifetime.

In the present paper we use the method of characteristic equations\(^8\) to solve in the general way the set of coupled differential equations describing the stimulated Brillouin backscattering including optical absorption (which was derived from the Maxwell and Navier–Stokes equations considering the optical gain higher than the acoustical one).

The expressions of pump- and Stokes-wave intensities obtained by us for an arbitrary time dependence of the pump pulse have been proved to be more accurate in comparison with the previous results (Kaiser and Maier,\(^1\) Kroll\(^2\)) and were derived by consideration of the first-order time and spatial derivatives for both cases: a pump pulse shorter and longer than the phonon lifetime.

The compression ratio in the same above-defined regime is also calculated. We redefined the steady-state regime of SBS in the saturation region of our solutions as a function of the pump-wave intensity, and this definition also provided a sufficient condition for a stationary process. The theoretical results were checked experimentally in a conventional configuration for SBS with a YAG:Nd laser and carbon disulphide as a nonlinear material. There is a good agreement between experimental and theoretical results.

2. SOLVING STIMULATED-BRILLOUIN-SCATTERING EQUATIONS USING THE CHARACTERISTIC EQUATIONS

The SBS process can be modeled by a set of differential equations describing the interaction between a light wave (electrical field) and an acoustical wave yielded by the former one in a nonlinear optical material.\(^9\) To solve analytically this set of nonlinear coupled equations, we have used the integration on characteristic equations,\(^8\) a method that leads to more general solutions (analytical distributions in time, \(t\), and space, \(z\), for an arbitrary temporal shape of the optical pump pulse).

With the expressions defined in Refs. 1 and 10–13 for the optical and acoustical gains, the condition for an optical gain higher than the acoustical gain may be written in the form

\[
(\Delta \rho)^2 \approx \frac{\gamma^e}{2cn} (I_L I_s)^{1/2},
\]

where \(\Delta \rho\) is the density variation of the nonlinear medium, \(v\) is the velocity of the acoustical waves, \(\gamma^e\) is the electrostrictive coefficient, \(n\) is the refractive index, \(c\) is the speed of light, and \(I_L, I_s\) are the intensities of the pump and the scattered fields.

In the case that the Navier–Stokes equation is purely deterministic (we neglect thermal effects, the induced turbulence, multiplicative ionization effects, and the hydrodynamic shock waves) the phase relation holds:

\[
\epsilon = \varphi_L - \varphi_S - \varphi_f = 0,
\]

where \(\varphi_L, \varphi_S, \) and \(\varphi_f\) are phases associated with the optical pump, the optical scattered, and the acoustical fields.

The acoustical field can be considered approximately stationary and can be substituted into the Maxwell equations, leading to
\[
\frac{n}{c} \frac{\partial I_{L_c}}{\partial t} + \frac{\partial I_{L_c}}{\partial z} = -\alpha I_{L_c} - g_B^* I_{L_c} I_S,
\]
\[
\frac{n}{c} \frac{\partial I_S}{\partial t} - \frac{\partial I_S}{\partial z} = -\alpha I_S + g_B^* I_{L_c} I_S,
\]

where \( \alpha \) is the linear optical loss in the material and \( g_B^* \) is the optical gain associated with the SBS process. The system of Eqs. (3) was obtained from Maxwell and Navier\textendash Stokes equations by use of a method indicated by Kaiser and Maier\textsuperscript{1} and conditions (1) and (2). It neglects the second-order derivatives with respect to the first-order ones. More general equations than were described in Refs. 1 and 12 are now derived.

In the space \((z, t)\) the characteristic equations associated with system (3) have the form\textsuperscript{8} [Fig. 1(a)]

\[\xi_L = \frac{c}{n} t + z,\]

\[\xi_S = \frac{c}{n} t + z_c - z.\]

The initial conditions for the system of Eqs. (3) are [Fig. 1(b)]

\[I_{L_c}(z, t)|_{z=0} = I_{L_0}(t),\]
\[I_S(z, t)|_{z=z_c} = I_{S_0}(t),\]

where \( I_{L_c}(t) \) is the time dependence of the pump pulse at the entrance of the cell, \( I_{S_0} \) is the spontaneous Stokes intensity, and \( z_c \) is the maximum interaction length between the optical and acoustical fields within the nonlinear medium.

We define

\[z_c = \begin{cases} \frac{ct_L}{n}, & t_L < \tau \\ \frac{c\tau}{n}, & t_L > \tau \end{cases},\]

Fig. 1. (a) Characteristics lines \( \Gamma_1 \) and \( \Gamma_2 \) associated with equation system (3); \( z_c \) and \( t_L \) are defined by Eq. (6); \( Q \) is the intersection of the Stokes wave front with the time axis at \( z = 0 \); \( L \) is the intersection of the pump wave front with the time axis at \( z = 0 \); \( P \) is the intersection of the Stokes and pump wave fronts. (b) The dashed volume represents the intersection region between the optical and the acoustical fields. \( I_{L_0} \) is the intensity of the pump pulse defined at \( z = 0 \); \( I_{L_c} \) is the intensity of the pump pulse defined at the exit of the interaction region, \( z = z_c \); \( I_S \) is the intensity of the Stokes field. (c) The surface \((QP_1L_1)\) is the domain of interaction when \( t_L < \tau \) and \( z_c = c\tau/n \); \( \Gamma_1 \) and \( \Gamma_2 \) are the light wave fronts; \( \Gamma_3 \) is the acoustical wave front; \( P_2L_3 \) is the pump wave front at \( z = c\tau/n \). (d) The surface of triangle \((QP_2L_2)\) represents the domain of interaction when \( t_L > \tau \) and \( z_c = c\tau/n \).
where \( t_L \) is the duration of the pump pulse and \( \tau \) is the phonon lifetime in the nonlinear medium.

The definition of \( z_c \) is related to the interaction time of the pump (light) and the acoustical waves, in two cases:

1. When \( t_L < \tau \), the interaction time is given by the pump-pulse duration, because after the pump pulse ends, there is nothing to be compressed even though the acoustical wave is still present. This situation is shown in Fig. 1(c), where the surface of the triangle \( QP_2L_2 \) represents the interaction (superimposition) of the three fields (pump, acoustical, and Stokes).
2. When \( t_L > \tau \), the interaction time is determined by the phonon lifetime, \( \tau \), because the long light interaction with the nonlinear medium is more affected by instabilities (thermal and hydrodynamic fluctuations) of the acoustic phonons from the lowest light intensities.\(^{14}\) We assume that the coherence of the conjugated beam is determined by random processes with lifetimes of the order of the phonon ones (similarly with the previous case). This situation is illustrated in Fig. 1(d), where the surface of triangle \( QP_1L_1 \) represents the interaction domain of the three fields.

From relations (3) and (6) we obtain the derivatives in function of \( \xi_L \) and \( \xi_S \):

\[
\frac{\partial}{\partial \xi_L} = \frac{1}{2} \left[ \frac{n}{c} \frac{\partial}{\partial t} + \frac{\partial}{\partial z} \right],
\]

\[
\frac{\partial}{\partial \xi_S} = \frac{1}{2} \left[ \frac{n}{c} \frac{\partial}{\partial t} - \frac{\partial}{\partial z} \right].
\]

In order to get a simple parametric representation of system (3) we write the derivatives as a function of \( \partial / \partial \xi_S \), in the form

\[
\frac{\partial}{\partial \xi_S} = \frac{1}{2} \left[ \frac{n}{c} \frac{\partial}{\partial t} + \frac{\partial}{\partial (z_c - z)} \right].
\]

In order to calculate the duration of the Stokes pulse, we choose the following form for the optical pump pulse:

\[
I_L(t) = I_0 f(t),
\]

where \( I_0 \) is the maximum intensity of the optic pump pulse and the envelope function \( f(t) \) is continuous, together with its derivatives up to the second one, with

\[
0 < f(t) \leq 1 \quad \text{if} \quad t \in [0, t_L]
\]

\[
f(t) = 0 \quad \text{elsewhere.}
\]

If the pump pulse is steplike, the scattered pulse will be different. Thus the scattered pulse has a different shape than that of the pump pulse, but is dependent on the shape of the pump pulse.

We calculate the duration of the Stokes pulse at 1/e from the maximum value of the Stokes intensity, \( t_S \), for the particular shape of the pump pulse:

\[
\begin{align*}
I_L(t) &= I_0 \left[ I_{L_0}(t) + I_{S_0}(t) \right] \exp(-az) \\
&= I_{L_0}(t) + I_{S_0}(t) \left[ \exp\left( g_B^p I_{L_0}(t) + I_{S_0}(t) l(z) \right) \right],
\end{align*}
\]

\[
\begin{align*}
I_S(t) &= I_{S_0} \left[ I_{L_0}(t) + I_{S_0}(t) \right] \exp(-az) + g_B^p I_{L_0}(t) + I_{S_0}(t) l(z) \\
&= I_{L_0}(t) + I_{S_0}(t) \left[ \exp\left( g_B^p I_{L_0}(t) + I_{S_0}(t) l(z) \right) \right].
\end{align*}
\]

Where \( l(z) = [1 - \exp(-az)]/\alpha \).
\[ I_{L_0} = I_0 \left[ 2 \frac{2t}{I_L} - \frac{2t^2}{I_L^2} \right]. \] (18)

The temporal width obtained by Hon\(^{3,16}\) for the scattered pulse using the expression for the special value \(t_c = \frac{ct_L}{n}\)

deduced for the pump pulse is a particular case of our formula (19), which can be deduced for the special value \(\alpha = 0\).

From relations (14) and (19) we obtain the compression ratio \(\left(\frac{t_L}{t_S}\right)\):

\[
\left( \frac{t_L}{t_S} \right)_{t_L < \tau} = \begin{cases} \left[ g_B^5 I_0 \left( \frac{c\tau}{n} \right) \right]^{1/2} & \text{for } g_B^5 I_{L_0}(z_c) - G < 0 \\ 1 & \text{for } g_B^5 I_{L_0}(z_c) - G > 0 \end{cases}
\] (20)

Similar to relation (20), relation (14) may be written in the form

\[
\left( \frac{I_S(t)}{I_{L_0}(t)} \right)_{t_L < \tau} = \begin{cases} \exp(-\alpha z_c + g_B^5 I_{L_0}(z_c) - G) & g_B^5 I_{L_0}(t) - G < 0 \\ \exp(-\alpha z_c) & g_B^5 I_{L_0}(t) - G > 0 \end{cases}
\] (21)

From relations (20) and (21) one can observe that for small pump intensities the Stokes intensity is proportional to the pump intensity and the compression ratio has high values.

For high pump intensities a saturation process appears for the Stokes intensity and the compression ratio becomes unity. In this region there is no compression \((t_L/t_S \to 1)\) and the Stokes intensity is

\[ I_S(t) = I_{L_0}(t)\exp(-\alpha z_c) \quad (t_L < \tau). \] (22)

We suggest calling this regime a stationary regime, as an extension of the definition given by Kaiser and Maier\(^1\) and Kroll\(^2\) and commonly used in the SBS literature. In this regime the temporal dependencies of the scattered and the pump pulses are similar up to the multiplicative constant \(\exp(-\alpha c t_L/n)\).

When the duration of the pump pulse is longer than the phonon lifetime \((t_L > \tau)\) and the pump intensity is high, using the expression for \(z_c\) from Eq. (6), we obtain

\[
I_S(t) = \frac{I_{L_0}(t)\exp\left[ -\alpha \frac{c\tau}{n} + g_B^5 I_{L_0}(t)\left( \frac{c\tau}{n} \right) - G \right]}{1 + \exp\left[ g_B^5 I_{L_0}(t)\left( \frac{c\tau}{n} \right) - G \right]},
\] (23)

a similar expression to that of Eq. (14).

Taking the similar shape for the pump pulse as that in Eq. (18), the duration of the Stokes pulse as in

\[
t_s = \left[ \frac{t_L}{g_B^5 I_0 \left( \frac{c\tau}{n} \right)^{1/2}} \right].
\] (24)

In this case the compression ratio \(\left(\frac{t_L}{t_S}\right)_{t_L > \tau}\) is

\[
\left( \frac{t_L}{t_S} \right)_{t_L > \tau} = \begin{cases} \left[ g_B^5 I_0 \left( \frac{c\tau}{n} \right) \right]^{1/2} & g_B^5 I_{L_0}(t)\left( \frac{c\tau}{n} \right) - G < 0 \\ 1 & g_B^5 I_{L_0}(t)\left( \frac{c\tau}{n} \right) - G > 0 \end{cases}
\] (25)

In both cases, \(t_L > \tau\) or \(t_L < \tau\), we redefined the steady-state regime of SBS in the saturation region of our general solutions (14) and (23), which are dependent on the pump-pulse intensity and duration and on the absorption of the nonlinear medium. The usual definition of this regime, implying the cancellation of the temporal derivatives\(^1\) leads, in our opinion, to an oversimplification of the process evolution and to an insufficient condition for the existence of such a steady-state regime.

From Eqs. (20) and (25) we can deduce the condition for pulse compression \((t_L/t_S > 1)\). In the case \(t_L > \tau\) and \(\alpha = 0\) this condition is

\[ G \geq g_B^5 I_0 \frac{c\tau}{n}. \] (26)

In the case \(t_L < \tau\) and \(\alpha = 0\) the pulse compression appears when

\[ G \geq g_B^5 I_0 \frac{ct_L}{n}. \] (27)

One can remark that the compression conditions, obtained in Eqs. (26) and (27), are similar up to the times involved, \(\tau\) and \(t_L\), respectively. They are valid only for pump pulses as in Eq. (18).

4. INFLUENCE OF STOCHASTIC PROCESSES ON THE SOLUTIONS OF THE STIMULATED BRILLOUIN SCATTERING EQUATIONS IN THE CASE WHEN THE DURATION OF THE PUMP PULSE IS LONGER THAN THE PHONON LIFETIME \((t_L > \tau)\)

When the duration of the pump pulse is longer than the phonon lifetime, \(t_L > \tau\), and the pump intensity is high, the acoustical wave virtually does not exist and the scattering will take place on the fluid disturbances.\(^{14,15}\) The equations for the SBS process could be written in the form
scattering. The rigorous analysis (the solution of the nonlinear medium appear other nonlinear processes with variable plest case for this modeling is the case when the random From the classical theory of the optical coherence the de-

\[ \frac{n}{c} \frac{\partial I_{L_e}}{\partial t} + \frac{\partial I_{L_s}}{\partial z} = -\alpha I_{L_e} - (1 - \epsilon') g_B^2 I_{L_e} I_{S}, \]

\[ \frac{n}{c} \frac{\partial I_{S}}{\partial t} - \frac{\partial I_{S}}{\partial z} = -\alpha I_{S} + (1 - \epsilon') g_B^2 I_{L_e} I_{S}, \] (28)

where \( \epsilon' \) becomes a random-phase variable. The simplest case for this modeling is the case when the random variable \( \epsilon' \) has a Gaussian distribution, with zero mean and dispersion \( \sigma \). The random variable \( \epsilon' \) has the following properties (Gaussian process):

\[ \langle \epsilon'(\eta) \rangle = 0, \]

\[ \langle \epsilon'(\eta) \epsilon'(\eta') \rangle = 2 \sigma^2 g_B I \delta(\eta - \eta'), \] (29)

where \( \langle \rangle \) means the average, \( \delta(\xi - \xi') \) is the Dirac function, and \( \sigma \) is the dispersion of the Gaussian process. From the classical theory of the optical coherence the degree of coherence has, from the mathematical point of view, the significance of the dispersion of the stochastic process.

In our paper the square of the dispersion \( (\sigma^2) \) of the stochastic process can be interpreted as an effective interaction length that can be smaller or higher than the maximum coherence length \( (c \tau/n) \). In this case the phase relation holds:

\[ \epsilon' = \varphi_L - \varphi_S - \varphi_d, \] (30)

with \( \varphi_d \) as the disturbance process.

When \( t_e > \tau \) and the pumping intensity is high, in the nonlinear medium appear other nonlinear processes with random character (great fluctuations of the thermal field, induced turbulence, multiplicative ionization phenomena, shock hydrodynamic waves, etc.), besides the Brillouin scattering. The rigorous analysis (the solution of the Navier–Stokes equation in the complete form) of the influence of all factors that disturb the SBS process is very difficult.

Consequently, we shall use the method of separation of the slow variables against the rapid ones (through which the disturbing process shown before may be treated like a Gaussian process).

The steps in this mathematical procedure pass through the building of the statistical Liouville equation (SLE) in the intensity space, then the obtaining of Fokker–Planck–Kolmogorov equation (by statistical Liouville equation averaging) and finally, the deduction of the evolution equations for the mean values of the intensities, \( I_{L_e} \) and \( I_{S} \). In this manner one can obtain a set of deterministic equations that describes the evolution of the mean values of the intensities on the characteristic lines:

\[ \frac{n}{c} \frac{\partial I_{L_e}}{\partial t} + \frac{\partial I_{L_e}}{\partial z} = -\alpha I_{L_e} - g_B^2 I_{L_e} I_{S} - \alpha^2 (g_B^2)^2 (I_{L_e} I_{S}^2 - I_{L_e}^2 I_{S}), \]

\[ \frac{n}{c} \frac{\partial I_{S}}{\partial t} - \frac{\partial I_{S}}{\partial z} = -\alpha I_{S} + g_B^2 I_{L_e} I_{S} - \alpha^2 (g_B^2)^2 (I_{L_e} I_{S}^2 - I_{L_e} I_{S}^2). \] (31)

The process of averaging Eqs. (28) is shown in Appendix A.

The results of these calculations show that the influence of the random perturbations of the SBS process produce another diffusion process characterized by \( \alpha^2 (g_B^2)^2 \), that limits, in intensity and duration, the scattering process.

The initial conditions of system (28) are identical with those from relations (5). Using the parametric representation from Eqs. (8) and (9), the system of Eqs. (28) becomes

\[ \frac{\partial I_{L_e}}{\partial \eta} = -\alpha I_{L_e} - g_B^2 I_{L_e} I_{S} - \alpha^2 (g_B^2)^2 (I_{L_e} I_{S}^2 - I_{L_e}^2 I_{S}), \]

\[ \frac{\partial I_{S}}{\partial \eta} = -\alpha I_{S} + g_B^2 I_{L_e} I_{S} - \alpha^2 (g_B^2)^2 (I_{L_e} I_{S}^2 - I_{L_e} I_{S}^2). \] (32)

Summing up Eqs. (32), we obtain a linear equation:

\[ \frac{\partial}{\partial \eta} (I_{L_e} + I_{S}) = -\alpha (I_{L_e} + I_{S}) \] (33)

with the solution

\[ \frac{\partial}{\partial \eta} (I_{L_e} + I_{S}) = -\alpha (I_{L_0} + L_S). \] (34)

From the initial conditions [Eq. (4)],

\[ c_1 = I_{L_0}(t) + I_{S_0} \sim I_{L_0}(t). \] (35)

By separating the variables in the autonomous system of Eqs. (32), we obtain a system of equations with separate variables, \( I_{L_e}, I_{S} \), but nonautonomous:

\[ I_{L_0}^2 \frac{\partial I_{L_e}}{\partial \eta} = I_{L_0}^2 - Q_1 I_{L_e}^2 + Q_1 I_{L_e}, \]

\[ I_{S_0}^2 \frac{\partial I_{S}}{\partial \eta} = I_{S_0}^2 - h_2 I_{S}^2 + h_1 I_{S}, \] (36)

where

\[ \eta' = -2 \eta (\sigma g_B^2 I_{L_0})^2, \]

\[ Q_1 = \frac{1}{2} I_{L_0} \exp(-\alpha \eta) + \frac{1}{\sigma g_B^2} \exp(-\alpha \eta), \]

\[ Q_2 = \frac{3}{2} I_{L_0} \exp(-\alpha \eta) + \frac{1}{3 \sigma g_B^2} \exp(-\alpha \eta), \]

\[ h_1 = \frac{1}{2} I_{L_0} \exp(-\alpha \eta) - \frac{1}{3 \sigma g_B^2} \exp(-\alpha \eta), \]

\[ h_2 = \frac{3}{2} I_{L_0} \exp(-\alpha \eta) - \frac{1}{3 \sigma g_B^2} \exp(-\alpha \eta). \] (37)

The nonlinear differential equations from Eqs. (36) do not accept prime integrals; thus we cannot obtain exact solutions by analytical methods because they are nonautonomous. We suggest, however, an analytical treatment of system (36) within the following approximation:

\[ \frac{d \eta}{d \eta'} = \frac{1}{2(\sigma g_B^2 I_{L_0})^2} \approx \frac{ct_L}{n}. \] (38)
The physical significance of inequality (38) is the following: we notice that variable \( \eta' \) is more rapid than variable \( \eta \). The result is that the functions defined in relations (37), \( (Q_{1,2}, h_{1,2}) \), can be considered constant against the variable \( \eta' \). Moreover, from relations (37), the functions \( (Q_{1,2}, h_{1,2}) \) are slowly varying with \( \eta \), when \( \alpha < 2(\sigma g_B I_{L_0})^2 \). (39)

Inequality (39) shows that the limitation of the SBS process owing to the random fluctuations (Gaussian process) is stronger than the limitation owing to the linear losses of the optical fields (pump and Stokes).

Equations (36) can be integrated directly by use of initial conditions (5), to lead to the following prime integrals for \( I_{L_c} \) and \( I_S \):

\[
I_{L_c} \bigg| I_{L_c} - \frac{1}{2} \left( I_{L_0} + \frac{1}{\alpha^2 g_B^2} \right) 2 g_B^2 g_B^2 I_{L_0} \frac{1}{1-\alpha^2 g_B^2 I_{L_0}} \right. \\
\times \exp\left[ -\left( \frac{\alpha^2 g_B^2 I_{L_0}}{1-\alpha^2 g_B^2 I_{L_0}} \right) \eta \right], \\
I_S = c_3 I_S - I_{L_0} \frac{1}{1-\alpha^2 g_B^2 I_{L_0}} \left( 2 g_B^2 g_B^2 I_{L_0} \frac{1}{1-\alpha^2 g_B^2 I_{L_0}} \right) \exp\left[ -\left( \frac{\alpha^2 g_B^2 I_{L_0}}{1-\alpha^2 g_B^2 I_{L_0}} \right) \eta \right],
\]

(40)

where \( c_2, c_3 \) are integration constants.

In the limit \( \alpha^2 \to 0 \), relations (40) are identical to relations (14) and (15).

For a large optical gain and for a small dispersion \( (\sigma^2) \), solutions (40) become:

\[
I_{L_c}(t) = \frac{I_{L_0}(t) \exp \left[ (g_B^2 I_{L_0} - \alpha - 2 g_B^2 g_B^2 I_{L_0}^2) \frac{c \tau}{n} - G \right]}{1 + \exp \left[ (g_B^2 I_{L_0} - \alpha - 2 g_B^2 g_B^2 I_{L_0}^2) \frac{c \tau}{n} - G \right]},
\]

\[
I_S(t) = \frac{I_{L_0}(t)}{1 + \exp \left[ (g_B^2 I_{L_0} - \alpha - 2 g_B^2 g_B^2 I_{L_0}^2) \frac{c \tau}{n} - G \right]}.
\]

(41)

We notice that solutions (41) differ from Eqs. (14) and (15) by a supplementary term in the exponential gain \( (\sigma^2 g_B^2 I_{L_0})^2 \), which can be interpreted as an additional diffusion process of the optical field. Similarly, we can calculate a compression ratio for \( t_L > \tau \):

\[
\lim_{\alpha^2 \to 0} \left| \frac{t_L}{I_S} \right| = \left| \frac{t_L}{I_{L_0}} \right| \frac{1}{t_L > \tau}.
\]

(43)

Thus the statistical modeling of the SBS process leads, in the limit \( \alpha^2 \to 0 \), to the same results as the conventional one.

From Eq. (42),

\[
\lim_{\alpha^2 \to 0} \left| \frac{t_L}{I_S} \right| = \lim_{\alpha^2 \to 0} \left| \frac{t_L}{I_{L_0}} \right| = 1 \left( \sigma^2 < \frac{1}{g_B^2 I_{L_0}}; \quad \sigma^2 > \frac{1}{2g_B^2 I_{L_0}} \right).
\]

(44)

The calculations of the conversion efficiency in the SBS process \( (E_S/E_L) \) in the two cases considered in this paper \( (t_L > \tau \) or \( t_L < \tau) \) are presented in Appendix A. These calculations show that the conversion efficiency \( (E_S/E_L) \) differ in the two cases owing to the value of the compression ratio \( (t_L/I_S) \).

5. EXPERIMENTAL SETUP AND RESULTS

The theoretical results were checked experimentally in a conventional configuration for SBS by use of a YAG:Nd laser (oscillator–amplifier system) and carbon disulphide in a glass cell as the nonlinear material.

The laser oscillator was operated in the Q-switched mode with a LiF:F2 crystal with the initial transmission of 18%. The oscillator was kept near threshold in order to generate a single pulse with a pump energy of 15 J. The resonator length is 40 cm. The short resonator and the Q switch with a small initial transmission allowed the generation of a pulse of 8-ns duration. A stack of four glass plates (Brewster angle) was used to get a linearly polarized output. To achieve the transverse-mode selection, we used an internal aperture of 1.5-mm diameter.

The output pulse energy of the oscillator was amplified, by a single pass, in a Nd:YAG module. The output pulse energy of the oscillator–amplifier laser system was 40 mJ.

The amplified pulse was focused with a convergent lens into a cell containing CS2 as nonlinear medium. Between the amplifier and the cell, an optical isolator (with a Glan prism and a Fresnel rhombus) is introduced.

The energy incident on the cell and the energy backscattered from the cell are measured with a TRG calorimeter. The temporal laser beam evolution was recorded by a fast photodiode (ITT) (rise time 60 ps) and a Tektronix 519 oscilloscope.

With the experimental system presented in Fig. 2 we were able to make systematic pulse-compression measurements in the Brillouin scattering, for \( t_L > \tau \).

For CS2 we used the parameters \( g_B^2 = 0.06 \text{ cm/MW} \) and \( \tau = 6 \text{ ns} \), at \( \lambda = 1.06 \mu \text{m} \). The SBS cell used in our experiment was long enough (1.2 m) in order to allow the
interaction length given by Eq. (6), when the laser pump pulse width is larger than the phonon lifetime.

In our experiments we used a long focal length of the lens (1 m) in order to have a reduced focusing and to be able to compare the experimental results with the calculations done for a plane wave.

When the laser pump-pulse width is larger than the phonon lifetime, the dependence of the energy-conversion efficiency ($E_S/E_L$) on the pump energy, for different values of the pump-pulse duration [Eq. (A.5) from Appendix A] is shown in Fig. 3. The stationary regime, defined by us in Eqs. (14) and (23), appears in these graphs for pump energies larger than 20 mJ.

In the same regime the dependence of the pulse-compression ratio ($t_L/t_S$) on optical pump energy is shown in Fig. 4. As was observed also, in previous experiments, the ratio ($t_L/t_S$) is small (maximum 8, for a pump energy of 40 mJ), increasing and saturating with the pump energy. From the experimental data the attenuation coefficient of CS$_2$ was deduced by a best fit as $\alpha = 0.01$ cm$^{-1}$ (see both Figs. 3 and 4). The experimental results are in good agreement with our new theoretical predictions for $t_L$. The first approximation of our theoretical model did not succeed in explaining the value of the absorption coefficient, deduced from the experiment ($\alpha = 0.028$ cm$^{-1}$) and the experimental fluctuations of the Stokes pulse amplitude.

The dependence of the pulse-compression ratio against the incident laser intensity, derived by use of the stochastic formalism, is illustrated in Fig. 5, for different values of the dispersion, together with Hon’s curve. We note that the experimental data are very well fitted by the curve with dispersion $\sigma = 0.2$ (at $\alpha = 0.028$ cm$^{-1}$).

One can remark that the stochastic theory is valid even in our experimental conditions when the duration of the incident laser pulse is close to the lifetime of the acoustic phonons, $\tau$. The maximum compression ratio (the shortest compressed pulse) for a given duration of the pump pulse, $t_L$, and different dispersions $\sigma$, can be deduced from Fig. 6.

6. CONCLUSIONS

We have derived a new system of equations to describe the SBS interaction using an isentropic approximation only. Solving this system analytically by the method of characteristic equations, we derived more accurate solutions for pump and Stokes intensities. In our paper the
For the calculation of the conversion-efficiency ratio $E_S/E_L$, we use the following assumptions: the energy of the optical pump field ($E_{L0}$) and the energy of the Stokes field ($E_S$) are uniformly in the transversal section of the beams; the areas of both optical beams are equal ($A_L = A_S$). We define the expressions of the energy for the two fields (pump and Stokes) in the following way:

$$E_{L0} = A_L \int_0^{t_L} I_{L0}(t) \, dt,$$

$$E_S = A_S \int_0^{t_S} I_S(t) \, dt. \quad (A.1)$$

The conversion efficiency is defined in the form

$$R = \frac{E_S}{E_L} = \frac{\int_0^{t_S} I_S(t) \, dt}{\int_0^{t_L} I_{L0}(t) \, dt}. \quad (A.2)$$

From the expressions of $I_S(t)$ obtained in Eq. (14),

$$I_S(t) = I_S[I_{L0}(t)]. \quad (A.3)$$

In this case the conversion efficiency becomes

$$R = \frac{\int_{I_{L0}(0)}^{I_{L0}(t)} \frac{dI_{L0}}{dt} \, dt}{\int_0^{t_L} I_{L0}(t) \, dt}. \quad (A.4)$$

Relation (A.4) defines the conversion efficiency for an arbitrary form of the optical pump pulse. Using expressions (14) and (42) for the Stokes intensity and expression (18) for the pump intensity [$f(t) \approx 1$], the integrals from relation (A.4) accept primitives in the case $t_L > \tau$, and the explicit form of the conversion efficiency $R$ is given by

$$E_S = \frac{3 \exp(-\alpha z_C - G)}{4 (t_L/t_S)^2} \left[ \frac{\sqrt{\pi}}{2} \left( \frac{t_L}{t_S} \right)^2 - \frac{1}{2} \right] \times \left[ \exp \left[ \frac{t_L}{t_S} \right] - \frac{1}{2} \right] \times \left[ \exp \left[ \frac{t_L}{t_S} - 2 \right] \right]$$

$$+ \frac{1}{2} \left[ \frac{t_L}{t_S} - \frac{1}{2} \frac{t_L}{t_S} - 2 \exp \left[ 4 \left( \frac{t_L}{t_S} - 1 \right) \right] \right],$$

$$\frac{t_L}{t_S} \leq \sqrt{G} \exp(-\alpha z_C), \quad g_B I_{L0}(z_C) - G > 0, \quad (A.5)$$

where the compression ratio can be written as

$$\frac{t_L}{t_S} = [g_B I_{L0}(z_C)]^{1/2}. \quad (A.6)$$

From Eq. (A.5),

$$\lim_{t_L/t_S \to \sqrt{G}} \left( \frac{E_S}{E_{L0}} \right) = \frac{3 \sqrt{\pi}}{8} \left( \frac{t_S}{t_L} \right) \left[ \exp \left( \frac{t_L}{t_S} - \frac{1}{2} \right) \right] \times \exp(-\alpha z_C). \quad (A.7)$$
From Eqs. (A.6) and (A.7) one can see that the conversion efficiency becomes proportional to $t_L$, at high compression ratios and low losses, as was shown in Ref. 12. More generally, the optical gain depends on the compression ratio through the erf function.

REFERENCES